

# Systematic determination of absolute absorption cross-section of individual carbon nanotubes

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**Optical absorption is the most fundamental optical property characterizing light-matter interactions in materials and can be most readily compared with theoretical predictions. However, determination of optical absorption cross-section of individual nanostructures is experimentally challenging due to the small extinction signal using conventional transmission measurements. Recently, dramatic increase of optical contrast from individual carbon nanotubes has been successfully achieved with a polarization-based homodyne microscope, where the scattered light wave from the nanostructure interferes with the optimized reference signal (the reflected/transmitted light). Here we demonstrate high-sensitivity absorption spectroscopy for individual single-walled carbon nanotubes by combining the polarization-based homodyne technique with broadband supercontinuum excitation in transmission configuration. To our knowledge, this is the first time that high-throughput and quantitative determination of nanotube absorption cross-section over broad spectral range at the single-tube level was performed for more than 50 individual chirality-defined single-walled nanotubes. Our data reveal chirality-dependent behaviors of exciton resonances in carbon nanotubes, where the exciton oscillator strength exhibits a universal scaling law with the nanotube diameter and the transition order. The exciton linewidth (characterizing the exciton lifetime) varies strongly in different nanotubes, and on average it increases linearly with the transition energy. In addition, we establish an empirical formula by extrapolating our data to predict the absorption cross-section spectrum for any given nanotube. The quantitative information of absorption cross-section in a broad spectral range and all nanotube species not only provides new insight into the unique photophysics in one-dimensional carbon nanotubes, but also enables absolute determination of optical quantum efficiencies in important photoluminescence and photovoltaic processes.**

polarization microscope | homodyne detection | carbon nanotube optical spectroscopy | excitons in carbon nanotubes

Single-walled carbon nanotubes (SWNTs), a model one-dimensional nanomaterial system, constitute a rich family of structures (1). Each single-walled nanotube structure, uniquely defined by the chiral index (n,m), exhibits distinct electrical and optical properties (2–5). Quantitative information of SWNT absorption cross-section is highly desirable for understanding nanotube electronic structures, for evaluating quantum efficiency of nanotube photoluminescence (5, 6) and photocurrent (7–9), and for investigating the unique many-body effects in 1D systems (10–16). Despite its obvious importance, reliable experimental determination of nanotube absorption cross-section at the single-tube level is still challenging (17). Previous absorption measurements on ensemble nanotube samples only provide averaged behavior (18–20). Recent absorption studies of individual nanotubes, suffering from small absorption signals and/or slow laser-frequency scanning, cannot determine the absolute absorption cross-section and are limited in achievable spectral range (15, 21–23).

We demonstrate here a high-sensitivity polarization-based homodyne method to measure nanotube absorption spectra. By manipulating the light polarization, we enhanced the nanotube-induced transmission contrast,  $\Delta I/I$ , by two orders of magnitude, and this enhanced transmission contrast can be quantitatively related to nanotube absorption cross-section along and perpendicular to the nanotube axis. Using this polarization control together with supercontinuum laser source, we realized high-throughput and broadband absorption measurements at the single-tube level; combined with electron diffraction technique on the same tube, it enables absolute determination of absorption cross-sections of individual chirality-defined nanotubes, to our knowledge for the first time. We obtained quantitative absorption spectra of over 50 SWNTs of different chiralities, and established a phenomenological formula for absorption cross-sections of different nanotubes. The chirality-dependent nanotube absorption spectra reveal unique 1D photophysics in nanotubes, including a universal scaling behavior of exciton oscillator strength and of exciton resonance linewidth.

## Results

**Experimental Design.** In 2D monolayer graphene, a universal absorption of  $\sim 2.3\%$  in the visible and near infrared range was predicted and observed (24, 25). For 1D nanotube with light

## Significance

Determination of optical absorption cross-section is of central importance to understanding a material; however, its realization on individual nanostructures, such as carbon nanotubes, is experimentally challenging due to the small extinction signal using conventional transmission measurements. Here we develop a technique based on polarization manipulation to enhance the sensitivity of single-nanotube absorption spectroscopy by two orders of magnitude. We systematically determine absorption cross-section over broad spectral range at the single-tube level for more than 50 chirality-defined single-walled nanotubes. Our data reveals chirality-dependent one-dimensional photophysics through the behaviors of exciton oscillator strength and lifetime. We also establish an empirical formula to predict absorption spectrum of any nanotube, which provides the foundation to determine quantum efficiencies in important photoluminescence and photovoltaic processes.

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polarized along its axis, the typical absorption of a single nanotube is 2–3 orders of magnitude smaller, i.e., at  $10^{-4}$  to  $10^{-5}$ , due to its nanometer diameter compared with micrometer illumination beam size. This small signal is easily overwhelmed by intensity fluctuation of the light source, making it difficult to measure single-tube absorption with conventional transmission spectroscopy. However, realizing that conventional transmission measurement is just one form of homodyne detection, we can measure the nanotube absorption cross-section with high sensitivity by optimizing the homodyne process.

From the interferometric point of view, our technique is a special example of homodyne detection. This method has been recently extended to carbon nanotubes with manipulation of polarizations (26, 27). In a homodyne measurement, the detected light intensity change ( $\Delta I/I$ ) originates from the interference between the signal ( $E_s$ ) and local oscillator electric field ( $E_{LO}$ ). Therefore,

$$\frac{\Delta I}{I} = \frac{|E_{LO} + E_s|^2 - |E_{LO}|^2}{|E_{LO}|^2} = \frac{2|E_s|}{|E_{LO}|} \cos \phi \xrightarrow{\text{Transmission}} \frac{\Delta T}{T} = \frac{2|E_{NT}|}{|E_{in}|} \cos \phi = -\alpha, \quad [1]$$

where  $\phi$  is the relative phase between  $E_s$  and  $E_{LO}$ , and we have ignored the small  $|E_s|^2$  term. The equation on the right describes conventional nanotube transmission measurements, where  $E_s$  and  $E_{LO}$  are, respectively, the nanotube forward-scattering wave ( $E_{NT}$ ) and the unperturbed incident light ( $E_{in}$ ), and  $\alpha$  is the nanotube absorption. If we can control  $E_s$  and  $E_{LO}$  with precisely defined relations to  $E_{NT}$  and  $E_{in}$ , we can potentially greatly enhance optical detection contrast  $\Delta I/I$ , and from it determine the absolute nanotube absorption cross-section; this can be achieved through polarization manipulation as shown in Fig. 1A. Two nearly crossed polarizers (with a small deviation angle  $\delta$ ) were used to control the incident and outgoing light polarization, and two polarization-maintaining objectives were placed confocally between this polarizer pair. Suspended individual nanotubes

were positioned at the focus of the objectives and with an angle of  $45^\circ$  with respect to the first polarizer. This configuration varies both local oscillator  $E_{LO}$  and signal  $E_s$  in a precise manner, where local oscillator is greatly reduced by  $E_{LO} = E_{in} \sin \delta$ , and the signal is related to the initial nanotube field by  $E_s \approx (E_{NT}^{\parallel} - E_{NT}^{\perp})/\sqrt{2}$ , where  $E_{NT}^{\parallel}$  and  $E_{NT}^{\perp}$  are nanotube field along and perpendicular to the nanotube axis, respectively. With this polarization control, we can obtain the polarization-dependent nanotube absorption through the enhanced homodyne optical contrast directly using (SI Text, Determine the Nanotube Absorption Constants  $\alpha_{\parallel}$  and  $\alpha_{\perp}$  from the Homodyne Modulation Signal)

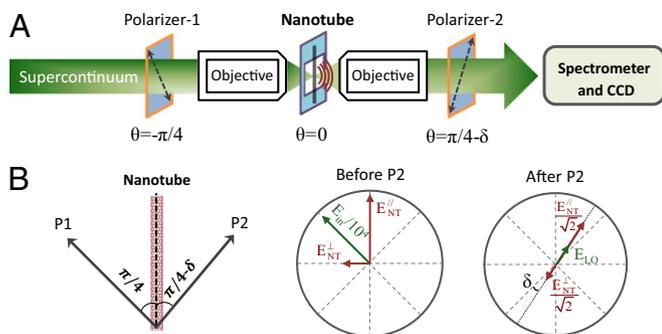
$$-\frac{\Delta I}{I}(\delta) = \frac{\alpha_{\parallel} - \alpha_{\perp}}{2} \text{ctg}(\delta) + \frac{\alpha_{\parallel} + \alpha_{\perp}}{2}, \quad [2]$$

where  $\alpha_{\parallel}$  and  $\alpha_{\perp}$  are nanotube absorption constants for light polarized parallel and perpendicular to the nanotube axis, respectively. Here,  $(\alpha_{\parallel} - \alpha_{\perp})/2$  characterizes the strong depolarization effect of nanotubes; it gives rise to a greatly enhanced optical contrast  $\Delta I/I$  at small  $\delta$ , which can reach  $\sim 1\%$  (compared with  $10^{-4}$  in conventional transmission change) and becomes easily detectable.

**Determination of Absolute Absorption Cross-Section.** The power of our technique is demonstrated in Fig. 2, which displays  $\Delta I/I$  spectra from a SWNT using different polarization settings. The chiral index of this nanotube was independently determined as (24,24) from its electron diffraction pattern (Fig. 2A and Fig. S1) (28). By gradually decreasing  $\delta$  (the deviation angle from the crossed polarizer position as labeled in Fig. 1A and Fig. S2) from  $5^\circ$  to  $0.5^\circ$ , the resulting modulation signal  $|\Delta I/I|$  increases steadily to 1% level at peak positions (Fig. 2B). Fig. 2C further shows that when  $\delta$  crosses zero, the local oscillator  $E_{LO}$  changes sign, as does the homodyne interference modulation  $\Delta I/I$ . This new polarization scheme, coupled with supercontinuum illumination and array detections, enables us to obtain single-nanotube  $\Delta I/I$  spectra across a wide spectral range in a few seconds, orders of magnitude faster than previous approaches (15, 21–23).

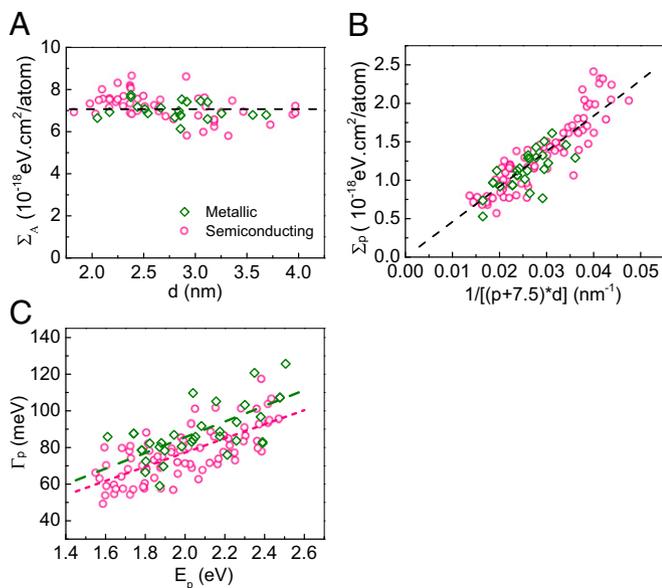
The spectra of  $\Delta I/I$  as a function of  $\delta$  allow direct determination of both  $\alpha_{\parallel}$  and  $\alpha_{\perp}$ . Fig. S3 displays  $\alpha_{\parallel}$  and  $\alpha_{\perp}$  over a broad spectral range for the (24,24) SWNT, which have values in the order of  $10^{-4}$  as expected. Nanotube absorption cross-sections are proportional to  $\alpha_{\parallel}$  and  $\alpha_{\perp}$ , and their absolute values can be obtained once the illumination beam size is known. We determine the focused supercontinuum beam profile at high accuracy by systematically measuring the absorption spectra with nanotubes at different positions in the focused beam (SI Text, Determine the Absolute Absorption Cross-Section from Absorption Constants). With this information, we obtained spectra of optical absorption cross-sections per atom along both parallel ( $\sigma_{\parallel}$ ) and perpendicular ( $\sigma_{\perp}$ ) polarizations for the (24,24) SWNT, which is displayed in Fig. 2D. We performed such absorption measurements on 57 chirality identified SWNTs, and it allows us to systematically examine polarization-dependent absorption cross-section in different nanotube species, to our knowledge for the first time. The  $\sigma_{\parallel}$  spectra show prominent and different exciton transition peaks in different nanotubes. These chirality-dependent exciton transitions provide rich information on chirality-dependent nanotube photophysics, as we describe later. The  $\sigma_{\perp}$  spectra, however, show a small and finite perpendicular absorption ( $\sim 1/4$  of the average  $\sigma_{\parallel}$  value), and they are largely featureless.

We first examine briefly the  $\sigma_{\perp}$  spectra, which have never been probed at the single nanotube level before. The small magnitude of  $\sigma_{\perp}$  can be attributed to depolarization effects (18, 19, 29, 30), but its lack of any resonance features is surprising at the first look. It is widely known that perpendicularly polarized light can excite transitions between adjacent cutting lines due to angular momentum selection rule (1). One naturally expects resonance



**Fig. 1.** Scheme of polarization-optimized homodyne detection for single-nanotube absorption. (A) Two nearly crossed polarizers (with a small deviation angle  $\delta$ ) were used to control the incident and outgoing light polarization, and two polarization-maintaining objectives were placed confocally between this polarizer pair. Suspended individual nanotubes were positioned at the focus of the objectives and with an angle of  $\pi/4$  with respect to the first polarizer polarization. We used broadband supercontinuum laser (spectral range 450–850 nm) as the light source and a spectrometer equipped with a silicon linear array for fast wide-spectral detection. (B) Interference between  $E_{in}$  and  $E_{NT}$  after the nanotube gives rise to nanotube extinction, whereas interference between  $E_s \approx (E_{NT}^{\parallel} - E_{NT}^{\perp})/\sqrt{2}$  and  $E_{LO} (=E_{in} \sin \delta)$  after polarizer 2 generates the final homodyne signal. Given that  $E_{LO}: E_{in} \sim 10^{-2}$  and  $E_s: E_{NT} \sim 1/\sqrt{2}$  with suitable polarizer setting, the enhanced homodyne modulation signal  $\Delta I/I$  can be two orders of magnitude higher than the transmission change  $\Delta T/T = -\alpha$ ; it enables high sensitivity measurements of nanotube absorption  $\alpha$ . The  $\delta$  angle is exaggerated for better visualization.





**Fig. 4.** Tube-dependent exciton oscillator strength and transition linewidth in 57 SWNTs. The diamonds and the circles represent data from 18 metallic and 39 semiconducting nanotubes, respectively. (A) Integrated absorption cross-section in the energy range of 1.55 to 2.48 eV. All of the data converge to the value of graphene integrated over the same energy region (dashed line). This convergence originates from an approximated f-sum rule. (B) Tube-dependent exciton oscillator strength is described by a universal scaling law of  $4.6 \times 10^{-24}/[(p+7.5) \cdot d]$  eV·cm<sup>2</sup> per atom (dashed line), where  $d$  is nanotube diameter and  $p$  is transition order indexing both semiconducting and metallic nanotube optical transitions. Metallic and semiconducting nanotubes, surprisingly, exhibit similar exciton oscillator strength, although they are characterized by significantly different electron-hole interactions. (C) Exciton transition linewidth in different nanotubes. On average, the linewidth increases linearly with the transition energies. The higher slope for metallic nanotube (long-dashed line) than that for semiconducting nanotube (short-dashed line) reveals (on average) slightly shorter exciton lifetime due to coupling to free electrons in metallic nanotubes. The large scattering present in the exciton linewidth suggests that ultrafast dynamics of the excited states can vary dramatically with the exact nanotube electronic structure.

heavyside step function,  $\Delta_p$  is the offset of continuum edge relative to the exciton transition, and the second term models the 1D density of states close to the band edge [*SI Text, Empirical Formula for Absorption Cross-Section of an (n,m) Nanotube*]. Red lines in Fig. 3 show that our phenomenological model nicely reproduces the experimental absorption spectra, with violet and dark yellow lines showing the exciton and continuum contribution, respectively. We note that the continuum absorption constitutes a significant portion of the total nanotube absorption oscillator strength, which has never been appreciated previously. The relative importance of continuum absorption increases with the nanotube diameter and the transition index  $p$ .

Exciton resonances, the most prominent features in the absorption spectra, encode rich 1D nanotube physics in their chirality-dependent behavior: (i) The exciton absorption oscillator strength provides new information on electron-hole interaction strength in the 1D nanotubes; (ii) the exciton transition linewidth, arising from the finite exciton lifetime, reveals the ultrafast relaxation dynamics of excited states; and (iii) the exciton transition energies and their dependence on nanotube species observed in absorption spectra here provide a valuable confirmation of the recent assignment established using Rayleigh scattering spectroscopy (16) with an accuracy of 10 meV.

The exciton takes its oscillator strength from band-to-band transitions due to the electron-hole correlation (12–15); its value depends on both the interband optical transition matrix element

and the exciton wave function at zero electron-hole separation. We observe in the experimental spectra (Fig. 3) that the exciton oscillator strength decreases significantly in large diameter nanotubes. A systematic examination reveals that the tube-dependent exciton oscillator strength can be described by a universal scaling law,  $\Sigma_p \sim 1/(p+7.5)d$ , as displayed in Fig. 4B. For semiconducting nanotubes, this scaling law can be understood theoretically using a model description of the excitonic effects, and the magnitude of observed exciton oscillator strengths agrees well with our theoretical predictions (32).

Fig. 4B also shows that the exciton oscillator strengths in metallic (diamonds) and semiconducting (circles) nanotubes have similar magnitude and fall on the same curve. This behavior appears surprising, because the electron-hole interactions are expected to be much weaker in metallic nanotubes due to free electron screening. Naively, the resulting exciton transition should have significantly smaller oscillator strength. Previous ab initio calculations (12, 13), however, showed that the oscillator strength from excitons in metallic nanotubes are comparable to those in semiconducting tube, although no systematic analysis was carried out. Further effort will be required to understand quantitatively the experimental data here, and it could lead to deeper insight into many-body interactions in 1D nanotubes.

**Transition Energy-Dependent Exciton Linewidth.** The optical linewidth of exciton transitions (for  $P > 1$ ) originates mainly from exciton lifetime broadening due to electron–electron and electron–phonon interactions. Ultrafast evolution of the exciton states plays a key role in important optoelectronic processes such as multi-exciton generation (9) and impact ionization (2). Our data provide a unique opportunity to investigate the chirality-dependent ultrafast dynamics in SWNTs. We plot in Fig. 4C the observed exciton linewidth in semiconducting (circles) and metallic (diamonds) nanotubes as a function of exciton transition energies; it shows that (i) the exciton linewidth, on average, increases linearly with transition energy and (ii) there is a large variation of exciton lifetime in different nanotube species.

The linear increase of average linewidth indicates a shorter exciton lifetime scales inversely with transition energy. This faster relaxation can be approximately attributed to increased phase space for electron–electron and electron–phonon scatterings at higher energy. Similar increases in excited-state relaxation rate with energy have been observed in studies of graphite (33) and low-order transitions in carbon nanotubes (34). The large variation of exciton lifetime, even for transitions at the same energy, indicates that the ultrafast electron relaxation may depend sensitively on the exact electronic structure, an important effect that to our knowledge has been previously unexplored; we hope our experimental data can stimulate their theoretical study to gain more insight on ultrafast electron dynamics in 1D systems.

#### Empirical Description of (n,m) Nanotube Absorption Cross-Section.

Last, we are able to develop an empirical formula for optical absorption spectra in all nanotubes (with diameter ranging from 1.2 to 3.2 nm) using the established relations for exciton oscillator strength and resonance width described above (Table S1). This phenomenological description reproduces the absorption spectra within 20% accuracy in our experimental energy region 1.45–2.55 eV [see the detailed description and example (Fig. S5) in *SI Text, Empirical Formula for Absorption Cross-Section of an (n,m) Nanotube*]. Such empirical formula of optical absorption cross-section for a wide variety of nanotubes across the visible spectral range will be a valuable reference for determining quantum efficiency of important optical processes such as photoluminescence and photovoltaics.

## Methods

**Sample Preparation and Chiral Index (n,m) Characterization.** In this study we combined single-tube absorption spectroscopy and electron-diffraction techniques on the same individual suspended nanotubes (SI Text, *Determine the Chiral Index and Absorption Spectrum of the Same Individual Carbon Nanotube*). The experimental scheme is illustrated in Fig. S1. Suspended long nanotubes were grown by chemical vapor deposition (CVD) across open-slit structures ( $\sim 30 \times 500 \mu\text{m}$ ) fabricated on silicon substrates. We used methane in hydrogen ( $\text{CH}_4:\text{H}_2 = 1:2$ ) as gas feedstock and a thin film ( $\sim 0.2 \text{ nm}$ ) of Fe as the catalyst for CVD growth at  $900 \text{ }^\circ\text{C}$  (35). This growth condition yields extremely clean isolated nanotubes free of amorphous carbon and other adsorbates. We determined the atomic structure of every nanotube from the electron diffraction pattern using nanofocused 80-keV electron beams in a JEOL 2100 transmission electron microscope (TEM) (28). By using the slit edges as spatial markers, the same individual nanotubes can be identified in both TEM and optical microscope setup.

**Optical Measurement.** Fig. 1A shows the scheme of the optical setup in our experiment; it includes a supercontinuum laser (Fianium SC-450), two Glan-Thompson polarizers (Thorlabs GTH10), two Mitutoyo Objectives (Plan APO 50x, N.A. 0.42; we used effective N.A. 0.3 in the experiment), an optical grating (300 lines per millimeter; Thorlabs GT150-03), a consumer camera (Nikon 5100) for imaging, and a 1D CCD array (Imaging Solution Group LW-ELIS-1024a-1394) for obtaining the spectrum.

The supercontinuum laser (470  $\sim$  850 nm) was used as our light source. The front objective serves to focus the supercontinuum light to the sample, and the second objective collects the nanotube scattering and transmission light. Polarizer 1 is placed in the incident beam with its transmission axis set  $45^\circ$  to the horizontal. Polarizer 2 is placed in the detection beam, and its polarization is controlled to be at a small angle  $\delta$  to the vertical direction of polarizer 1.

We obtained the homodyne modulation spectrum by taking two sets of spectra with the nanotube in ( $I_i$ ) and out ( $I_o$ ) of the supercontinuum focus. The integration time of each spectrum was 5 ms. The modulation signal was obtained as  $\Delta I/I = (I_i - I_o)/I_o$ . By shifting the nanotube in and out of the focus quickly, we can take a high-quality modulation spectrum within 2 s.

**Error Analysis.** In our study, we determined the absolute absorption cross-section of SWNTs for electrical field polarized along the nanotube direction using Eqs. S3 and S5, where

$$\alpha_{ij} = \frac{\Delta I/I(-\delta) - \Delta I/I(\delta)}{2} \tan(\delta) - \frac{\Delta I/I(\delta) + \Delta I/I(-\delta)}{2},$$

and

$$\alpha_{ij} = \sigma_{ij} \frac{\sqrt{2\pi}}{S_0} \frac{d}{R} \exp\left(-\frac{2(x-x_c)^2}{R^2}\right).$$

The final absorption cross-section  $\sigma_{ij}$  is obtained by fitting a curve of absorption constant  $\alpha_{ij}$  vs. nanotube position  $x$  based on the above equations, where the laser beam center  $x_c$  and diameter  $R$  are also fitting parameters.  $S_0$  is the area per carbon atom in graphitic lattice, and  $d$  is the nanotube diameter, which are discrete and exact values defined by the nanotube chiral index. The experiment uncertainty mainly arises from measurements of laser intensity change  $\Delta I$ , polarizer angle  $\delta$ , nanotube position  $x$ , and from systematic uncertainties of the technique, which we analyze in detail below.

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**Random Error from Measurements.** *Measurement uncertainty of  $I$  and  $\Delta I$ .* The measurement uncertainty of  $I$  and  $\Delta I$  is affected by laser fluctuation, detector noise, and photon shot noises. The resulting uncertainty in  $\alpha_{ij}$  can be obtained from statistical fluctuations in multiple measurements. In our studies, we integrated 300 spectra, which leads to a fluctuation of  $\alpha_{ij}$  value within 2% at the absorption peak and  $\sim 6\%$  for the nonresonant absorption. Fig. S3 shows a representative spectrum of  $\alpha_{ij}$ , where the high quality of the spectra confirms that laser fluctuation-induced uncertainty is within our estimated range.

**Determination of  $\delta$ .** We determine the polarization angle based on transmitted light intensity  $I_t$  after the second polarizer, which follows the equation  $I_t = I_{in} \sin^2(\delta)$ . In our experiment, we set  $\delta$  at 0.04, which corresponds to a transmission of  $1.6 \times 10^{-3}$ . The uncertainty of  $\delta$  arises from laser fluctuation and nonideal polarization extinction. The fluctuation of average laser intensity is less than 0.5%. The extinction ratio in our polarization microscope setup is  $4 \times 10^{-6}$ . Compared with the experimental transmission of  $1.6 \times 10^{-3}$ , the nonideal polarization leads to an uncertainty of 0.2% in  $\delta$ . Overall, the measurements of  $\delta$  adds an uncertainty of 0.5% in the value of  $\alpha_{ij}$ .

**Uncertainty of  $\sigma_{ij}$  from curve fitting of  $\alpha_{ij} = \sigma_{ij} \sqrt{2\pi}/S_0 \cdot \frac{d}{R} \cdot \exp(-2(x-x_c)^2/R^2)$ .** The uncertainties of  $\alpha_{ij}$  and position measurements contribute to the final uncertainty of through the curve fitting. Fig. S6 shows two representative fitting curves. From the fitting we estimate the uncertainty of  $\sigma_{ij}$  to be below 2–6%. The uncertainty of  $\sigma_{ij}$  shows a frequency dependence, and it is usually  $\sim 2\%$  at an absorption resonance and  $\sim 6\%$  for nonresonant absorption. This uncertainty in  $\sigma_{ij}$  is consistent with the fluctuation observed in the absorption cross-section spectra in Fig. 3.

**Systematic Error. Plane-wave assumption.** In our measurements we used focused light with an effective N.A. of 0.3, which does not have perfect in-plane linear polarization at the nanotube. The averaged in-plane field for N.A. = 0.3 is  $\sim 2\%$  smaller than an ideal plane wave; it leads to a systematic overestimation of nanotube absorption cross-section by 2% in our measurements.

**Error from neglecting higher-order terms.** In Eq. 1, we have neglected the higher-order term associated with  $|E_{\text{NT}}|^2/|E_{\text{LO}}|^2$ ; this yields an uncertainty  $\Delta\sigma/\sigma = \Delta\alpha/\alpha \sim E_{\text{NT}}/2E_{\text{LO}} < 1\%$ .

Based on the above analysis, our determination of nanotube absorption cross-section has an uncertainty of  $\sim 4\%$  at absorption resonances and an uncertainty of  $\sim 8\%$  for background absorption.

An independent check of our measurement uncertainty is the comparison of nanotube absorption with that of graphene, as shown in Fig. 4A; it has a SD of 8%. This comparison sets an upper limit of our measurement uncertainty at 8%, which is consistent with the uncertainty estimated above.

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